Energy-aware parallel task scheduling in a cluster

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**Highlights**
- This paper develops formal models for parallel tasks and a power aware cluster.
- Power aware scheduling for parallel tasks based on list scheduling is proposed.
- System model is based on SLA for green parallel task scheduling.
- The proposed methodologies are thoroughly investigated through simulations.

**ABSTRACT**
Reducing energy consumption for high end computing can bring various benefits such as reducing operating costs, increasing system reliability, and environmental respect. This paper aims to develop scheduling heuristics and to present application experience for reducing power consumption of parallel tasks in a cluster with the Dynamic Voltage Frequency Scaling (DVFS) technique. In this paper, formal models are presented for precedence-constrained parallel tasks, DVFS-enabled clusters, and energy consumption. This paper studies the slack time for non-critical jobs, extends their execution time and reduces the energy consumption without increasing the task's execution time as a whole. Additionally, Green Service Level Agreement is also considered in this paper. By increasing task execution time within an affordable limit, this paper develops scheduling heuristics to reduce energy consumption of a task's execution and discusses the relationship between energy consumption and task execution time. Models and scheduling heuristics are examined with a simulation study. Test results justify the design and implementation of proposed energy aware scheduling heuristics in the paper.

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1. Introduction

Nowadays, high end computing facilities can consume a very large amount of power, although they provide high performance computing solutions for scientific and engineering applications [1]. For example, operating a middle-sized data center (i.e., a university data center) demands 80,000 kW power [2]. It is estimated that computing resources consume around 0.5% of the world's total power usage [3], and if current demand continues, this is projected to quadruple by 2020. Energy consumption for high performance facilities thus contributes to a significant electric bill. Additionally, high power consumption in general results in higher cooling costs. Furthermore, to allow computing facilities to operate on high power for a long time will lead to a high temperature of computing systems, which further harms a system's reliability and availability. Therefore, reducing power consumption for high end computing becomes a critical research topic.

Modern processors are equipped with the Dynamic Voltage Frequency Scaling (DVFS) technique, which enables processors to be operated at multiple frequencies under different supply voltages. The DVFS technique thus gives opportunities to reduce the energy consumption of high performance computing by scaling processor supply voltages. Our research is devoted to developing...
scheduling heuristics which reduce energy consumption of parallel task execution by using the DVFS mechanism. A parallel task is a set of jobs with precedence constraints. Jobs in a parallel task may have some slack time for their execution due to their precedence constraints.

This paper makes a study of scheduling policies and application experiences to reduce power consumption of parallel tasks. Our first research issue is to minimize task execution time as well as reduce power consumption. The execution time of the non-critical jobs in a parallel task can be extended, thus giving an opportunity to scale down the supply voltages of processors. Based on the analysis of DVFS on non-critical jobs, we develop two power aware scheduling heuristics for parallel tasks, the Power Aware List-based Scheduling (PALS) algorithm and the Power Aware Task Clustering (PATC) algorithm.

Our second research objective is to make an study on energy and performance tradeoff for parallel task execution. The green Service Level Agreement (SLA) is introduced in this research. By negotiating with users via the green SLA, an energy-performance tradeoff algorithm is developed to reduce energy consumption with an affordable task execution time increase. We develop a simulation study on the proposed scheduling heuristics and make a performance evaluation.

We declare our contribution as follows:

- We develop formal models for parallel tasks and a power aware cluster and we also define the task scheduling issue.
- We develop two power scheduling heuristics for parallel tasks: the PALS and the PATC.
- We present the green SLA use scenarios and propose a new scheduling heuristics for energy aware parallel task scheduling, which makes a study of the tradeoff between the energy consumption and task execution time (performance).
- We build a simulation study and performance evaluation on the proposed heuristics. Test results justify our design and implementation of energy aware heuristics.

The rest of this paper is organized as follows. Section 2 introduces background and related work. Then Section 3 discusses the models for parallel tasks, DVFS, and compute clusters and Section 4 formally define the research issue of energy aware parallel task scheduling. Section 5 applies the DVFS technique on non-critical jobs of parallel tasks, which is the basis of the PALS and the PATC. We describe the scheduling heuristics of the PATC and the PALS in Sections 6 and 7. Section 8 presents the Service Level Agreement with performance metrics of green computing and proposes the research issue of energy-performance tradeoff for parallel task scheduling. Section 9 then presents the scheduling algorithm for the research issue proposed in 9. The complexity analysis for the proposed algorithms are presented in Sections 10 and 11 describes a simulation study on the proposed scheduling heuristics. Finally this paper is summarized in Section 12.

2. Related work

This section discusses background and related work of task scheduling, DVFS, and power aware cluster computing.

2.1. Parallel task scheduling

Task scheduling techniques in parallel and distributed systems have been studied in great detail with the aim of making use of these systems efficiently. Task scheduling algorithms are typically classified into two subcategories: static scheduling algorithms and dynamic scheduling algorithms. In static task scheduling algorithms, the task assignment to resources is determined before applications are executed. Information about task execution cost and communication time is supposed to be known at compilation time. Static task scheduling algorithms normally are non-preemptive—a task is always running on the resource to which it is assigned [4]. Dynamic task scheduling algorithms normally schedule tasks to resources in the runtime to achieving load balance among PEs, are based on the redistribution [5,6].

The list scheduling algorithm is the most popular algorithm in the static scheduling [7,8]. List based scheduling algorithms assign priorities to tasks and sort tasks into a list ordered in decreasing priority. Then tasks are scheduled based on the priorities. In this paper, we build a list based scheduling heuristic for parallel tasks—the PALS algorithm. The task execution information, such as task execution cost and communication cost, can be obtained by some profiling tools and compiler aides in advance.

The task graph clustering technique [9,10] is an effective static scheduling heuristic for scheduling parallel tasks. Given a task graph, “clustering” is the process of mapping task graph nodes onto labeled clusters. All tasks of the same cluster are executed in the same processor. In traditional task scheduling heuristics, the process of clustering tasks is an optimization of reducing the makespan of the scheduled graph. In this paper, we proposed the PATC algorithm, whose process of clustering tasks is guided by reducing the total power consumption of the scheduled graph.

2.2. Energy reduction via DVFS techniques

Dynamic voltage and frequency scaling (DVFS) has been proven to be a feasible solution to reduce processor power consumption [11,12]. By lowering processor clock frequency and supply voltage during some time slots, for example, idle or communication phases, large reductions in power consumption can be achieved with only modest performance losses. A DVFS-enabled cluster [1] is a compute cluster where compute nodes can run at multiple power/performance operating points. The DVFS techniques have been applied in the high performance computing fields, for example, in large data centers, to reduce power consumption and achieve high reliability and availability [13–15]. Popular DVFS-based software solutions for high end computing include:

- Scientific applications can be modeled with a Directed Acyclic Graph (DAG) and the critical path is identified for applications. Thus, it is possible to reduce energy consumption by leveling down the processor supply voltage during non-critical execution of tasks [16].
- Some work [17] builds online performance-driven runtime systems to automatically scale processor supply voltages.
- Some work applies DVFS during the communication phases of high performance computing, for example MPI [18,19].
- In addition to parallel applications, virtual machine scheduling can also use DVFS [1].

Our research in this paper falls into the first category: scheduling DAGs on multiple processors in a cluster with DVFS techniques.

2.3. Power aware task scheduling

A lot of work has developed DVFS for task scheduling. For example, Yao et al. [20] and Ali et al. [21] discuss scheduling independent tasks with DVFS on a single processor, Wei et al. [22] and Gruian et al. [23] use DVFS to schedule dependent tasks on multiple processors, Martin et al. [24] and Luo et al. [25,26] developed a power aware task scheduling algorithm for real time systems. As our work is devoted to developing power aware scheduling algorithms for dependent tasks, we compare our work with related research in this topic.

Zhang et al. [27], Martin et al. [24], Schmitz [28], and Luo et al. [26] schedule dependent tasks on real time, where the tasks...
3.2. Energy model

The energy consumption of modern processors for job execution, $\xi$, can be divided into two parts, dynamic energy consumption $\xi_{\text{dynamic}}$, and static energy consumption $\xi_{\text{static}}$ [36]. Static power consumption arises from running, bias, and leakage currents. Dynamic power consumption arises from the charging and discharging of the circuit node capacitances found on the output of every logic gate.

$$\xi = \xi_{\text{dynamic}} + \xi_{\text{static}}.$$  (3)

According to [37], the dynamic power consumption $P_{\text{dynamic}}$ is computed as follows:

$$P_{\text{dynamic}} = A \times C \times \delta \times f$$  (4)

where,

- $A$ is the percentage of active logic gates, which are charged dynamically;
- $C$ is the total capacitance load;
- $\delta$ is the supply voltage;
- $f$ is the processor frequency.

Then, we have:

$$\xi_{\text{dynamic}} = \sum_{\Delta t} P_{\text{dynamic}} \times \Delta t$$  (5)

Thereby, the whole power consumption could be estimated as follows:

$$\xi \propto \xi_{\text{dynamic}}.$$  (6)

In conclusion, we have the performance model:

$$\xi = \sum_{\Delta t} \left( \xi_{\text{static}} \times \xi_{\text{dynamic}} \times \Delta t \right)$$  (7)

where,

- $\delta$ is a constant determined by the PE.
- $\nu$ is the processor operating voltage during $\Delta t$.
- $f$ is the processor operating frequency during $\Delta t$.

3.3. Resource model

A compute cluster normally contains multiple compute nodes, which are formally termed as Processing Elements (PEs) in a context of parallel computing. This paper makes a study of homogeneous clusters: all PEs of the cluster have the same processing speed or provide identical processing performance in terms of MIPS (Million Instructions Per Second). A homogeneous cluster, $C$, contains $K$ PEs. The $k$-th PE $p_k$ has two properties:

- $p_k \cdot \nu \in V$ is the processor operating voltage
- $p_k \cdot f \in F$ is the processor operating frequency

$1 \leq k \leq K$ is the total number of PEs.

A cluster $C$ is defined by its set of processing elements

$$C = \bigcup_{1 \leq k \leq K} \{ p_k \}.$$  (9)

3.4. Parallel task model

A parallel task with precedence constrains is modeled as a Directed Acyclic Graph (DAG) $T = (J, E)$:

- $J$: a set of jobs (nodes in a DAG)
- $E$: a set of edges (links in a DAG)

$$J = \bigcup_{1 \leq n \leq N} \{ job_n \}.$$  (10)

where,

- $job_n$ is a job in the parallel task $J$.  

N is the total number of jobs. A job $j_n$ has 3 properties:
- weight is the instruction number of job $j_n$.
- $t^r$ is the starting time of job $j_n$.
- $t$ is the execution time of job $j_n$. If job $j_n$ is executed on PE, the job execution time is calculated as follows:

$$j_n \cdot t = \frac{job_n \cdot weight \times CPI}{PE \cdot F_{op}}$$

(11)

where, CPI is the number of cycles per instruction of PE, $F_{op}$ is determined by both the hardware and software of the cluster C, for example, computer architecture and instruction set (i.e., RISC or CISC), $job_n \cdot t^r$ is the job $j_n$'s execution time when PE is running with the maximum frequency $F_{max}$. Eq. (11) calculates job execution based on PE's operating frequency.

- $t^{end}$ is the end time of job $j_n$. We have:

$$job_n \cdot t^{end} = job_n \cdot t^r + job_n \cdot t.$$

(12)

Based on Eqs. (11) and (8), the energy consumption to execute job $j_n$ can be calculated as follows:

$$\varepsilon_n = \gamma \times v^2 \times job_n \cdot weight$$

(13)

where, $\gamma$ is a constant determined by the cluster and, irrelevant with the parallel task. $v$ is the PE supply voltage during the job $j_n$'s execution.

* $E$: a set of precedence constraints (edges in a DAG) $E$ defines partial orders (operational precedence constraints) on $J$. $e_i$ is an edge between job, and job, it means that job, must be completed before job, can begin, $1 \leq i, j \leq N$. $job_i, job_j \in J$. $e_i$ sometime can also be represented $job_i < job_j$.

$e_i$ has one property:

$$e_i \cdot cost \geq 0$$

is the amount of data required to be transferred from job, to job, $1 \leq i, j \leq N$. $job_i, job_j \in J$. Data are transferred from the PE where job, is executed to the PE where job, is executed.

As we are studying a homogeneous cluster, without loss of generality, $e_i \cdot cost$ can also be normalized as communication time. Now we discuss the relationship between $e_i \cdot cost$ and PE's operating frequency. It shows in [19] that the energy consumption and communication cost as processor frequency varies for four common MPI calls when different size of data are transferred among PEs. From the experiment results we can see energy can be saved up to 31% with at most a 5% communication time increase. In this paper, we ignore the communication time increase. In other words, when a PE's supplied voltage is scaled down, the data communication time remains unchanged.

### 4. Research problem definition

Here we firstly consider the best-effort scheduling research problem. Without damaging the performance of parallel task execution (task execution time), the best-effort scheduling algorithm tries to reduce the energy consumption for task execution.

Before we bring up the formal definition of the above research issues, the following term definitions are introduced.

- **TST**: Task Starting Time of $T$

$$TST = \min_{1 \leq n \leq N} job_n \cdot t^r$$

(14)

- **TFT**: Task Finish Time of $T$

$$TFT = \max_{1 \leq n \leq N} job_n \cdot t^{end}$$

(15)

- **makespan**: the schedule length of $T$

$$makespan = TFT - TST$$

(16)
6. The PATC algorithm

We summarize several obvious rules to guide the design of the PATC algorithm and the PALS algorithm.

1. Eq. (13) shows that given a certain task, a PE's supply voltage could be scaled down to a proper voltage to reduce the task's energy consumption. Certainly, this action may lead an increase of task execution time.

2. Research in [19] indicates that during the communication phase, the PE's supply voltage should be scaled down to the lowest level.

3. When a PE is idle (there is no task execution and data communication), its supply voltage should be leveled down to the lowest level.

This section presents the Power Aware Task Clustering (PATC) algorithm for parallel task scheduling. Traditional task clustering algorithm takes the following steps: (1) task clustering by zeroing edges, (2) cluster merging if the number of task clusters is greater than the number of PE, (3) task execution ordering in each task cluster, (4) each task cluster is allocated with a PE.

Traditional task clustering algorithm reduces the makespan by zeroing edges of high communication costs. Our Power Aware Task Clustering (PATC) algorithm, on the contrary, guides the edge zeroing process with objective of reducing power consumption. As shown in Algorithm 2, the PATC algorithm firstly marks all edges as unexamined and allocates each task a separate cluster. After sorting all edges in descending order of communication time, the PATC algorithm repeatedly merges tasks by zeroing the edges with high communication cost if the total power consumption is not increased. How to scale non-critical jobs' voltage and calculate the power consumption of a scheduled task graph have been discussed in Section 5.

Algorithm 2 The PATC algorithm

1 BEGIN
2 Initially all edges are marked unexamined and each task forms a separate cluster
3 Sort all edges in a descending order according to their communication costs
4 REPEAT
5 Zero the highest unexamined edge in the sorted list if the power consumption of the scheduled task graph does not increase
6 Mark the edge examined
7 When two clusters are merged, the tasks are ordered according to their b_level
8 UNTIL all edges are marked examined
9 END

Inside each cluster, tasks are executed in the order of their b_level. b_level is a normal priority assignment for jobs, which is defined as the length of a longest path from that job to the exit job. b_level is calculated with Algorithm 3.

7. The PALS algorithm

This section presents the Power Aware List-based Scheduling (PALS) algorithm for parallel tasks. The PALS algorithm (shown in Algorithm 4) firstly employs the ETE (Earliest Task First), a list-based scheduling algorithm (shown in Algorithm 5), to find the best-effort task response time for T. Then, it tries to reduce the energy consumption with the following methods:

- Scale down PE's voltages to a proper level, thus extending the execution time of the non-critical jobs without affecting the critical path.
- Scale the PE's voltage when it is idle or when it is in the data communication phase.
Algorithm 3 b_level calculation
1 BEGIN
2 r_list ← a list of all jobs job_i ∈ J sorted in a reversed partial order
3 Initialize all jobs in rtopo_list: b_level(job_i) ← 0
4 FOR each Job job_i ∈ rtopo_list DO
5 max_length ← 0
6 FOR each immediate succeeding job job_j of job job_i, DO
7 length ← b_level(job_j) + e_i,j cost
8 IF (length > max_length) THEN
9 max_length ← length
10 ENDIF
11 ENDFOR
12 b_level(T_i) ← job_i weight + max_length
13 ENDFOR
14 END

Algorithm 4 The PALS algorithm
1. schedule tasks via the ETF scheduling Algorithm 5
2. scale down PE’s voltages for all non-critical jobs with Algorithm 1

Given a parallel task T, the ETF algorithm [38,39] is described in Algorithm 5. Algorithm 5 allocates each job with a priority which can be calculated via different methods, for example, bottom level and top-level [40]. In our implementation, we use the bottom level. The bottom level of a node (job) in a DAG is the longest path beginning with the node and the top-level is the longest path reaching the node. The length of a path is defined as the sum of the weights of its nodes and edges. Then, Algorithm 5 selects ready jobs with the highest priority and schedules it on the PE with earliest task starting time.

Algorithm 5 The ETF scheduling algorithm
1. job_n.level: priority of task job_n ∈ J
2. ready_job_list: list of jobs that are ready to be executed
3. PE_list: list of PEs
4. pE_k avail: PE’s available time.
5 BEGIN
6 FOR each job job_i ∈ J DO
7 compute job_i level
8 ENDFOR
9 put all ready jobs into ready_job_list
10 sort all jobs job_n ∈ ready_job_list in decreasing order of job_n.level
11 put all PEs into PE_list
12 sort all PEs pE_k.t available = 0
13 REPEAT
14 IF (ready_job_list ≠ ∅) THEN
15 get a job job_n from ready_job_list
16 get a PE pE_k, which has the earliest available time pE_k.t available
17 schedule job_n on pE_k
18 arrange the communicate phase, calculate starting time and finish time of job_n on pE_k
19 delete the task from ready_job_list
20 update PE_list with increasing order pE_k.t available
21 ENDIF
22 update ready_job_list
23 UNTIL (every job job_i ∈ J has been scheduled)
24 END

8. SLA management for green computing

In previous sections, we make a study on reducing power consumption without increasing task execution time, which is termed as the “best-effort scheduling issue”. This section we analyze an interesting scenario: if a user is environmental respect and want to reduce power consumption by increasing its task execution time.

Green computing is a research topic to make computing with environmental concerns [41], for example, reduced energy consumption and reduced CO2 emissions. We develop power aware scheduling for parallel task in the context of green SLA (Service Level Agreement for Green Computing). Users can specify not only performance requirements for computing services, but users can also specify green computing requirements for executing their jobs. We define the green SLA in three phases:

- Green SLA contract definition.
  Our previous work [41] has summarized a number of green computing metrics, such as Data Center Infrastructure Efficiency (DCIE) [42,43], Power Usage Effectiveness (PUE) [43], Data Center energy Productivity (DCEP) [44], Space Watts and Performance (SWaP) [45], storage, network, and server utilization. The green SLA contract definition phase creates various green SLA templates based on above green computing metrics. Typical metrics includes task response time, CO2 emission, and power consumption. This phase also contains green SLA template publication and discovery.

- Green SLA negotiation and monitoring.
  Users develop their green SLA specification based on SLA templates and make a negotiation with computing resources, for example, a high performance cluster. Here are some examples of green computing service specifications:
  - Establish an execution service for x minutes if the total carbon emission of the service is below y tons.
  - I would like to accept z% task execution time increase to reduce w energy consumption.

- Green SLA enforcement.
  When a green SLA is reached, computing resources then execute the specified green services. For example, schedule tasks based on specified task execution time, CO2 emission and power consumption. We develop energy aware scheduling algorithms for parallel tasks based on user’s green SLA specifications.

Fig. 4 shows the conceptual framework for green SLA-based on energy aware scheduling in a cluster. Before a resource consumer submits a parallel job to a cluster, she/he firstly negotiates with a resource provider with normal performance metrics, like job response time, as well as with green metrics, for example, power consumption or CO2 emission. After an agreement is reached, the user then submits his/her job to the resource. The resource provider then schedules the incoming job to an energy aware cluster to guarantee the green metrics and computing performance.

With the green SLA negotiation, users agree to accept a tolerable performance loss, for example, additional 10% of task execution time, to reduce more energy consumption and make their computing more green. In contrast to the best-effort scheduling research problem, we term this research issue as the energy-performance tradeoff scheduling issue, whose main objective is to reduce energy consumption for task execution with an acceptable performance punishment.

The energy-performance tradeoff scheduling issue can be defined as: given parallel task T, a cluster C, and the schedule length makespan_{best}, of a best-effort schedule, find a feasible schedule which tries to minimize energy consumption by giving Task Execution Time makespan ≤ (1 + η) × makespan_{best}, η > 0 is the accepted task execution time extension, which is determined by the green SLA negotiation.


Now we discuss the energy-performance tradeoff problem: if a user agrees to tolerate an increase of his/her job execution time,
for example, $\eta$ of schedule length of the best-effort scheduling algorithm, how to schedule jobs to save more energy?

The energy-performance tradeoff algorithm is shown in Algorithm 6. It firstly gets the best-effort scheduling length via Algorithm 5. Then, it scales both the critical time slots in Algorithm 7 and non-critical time slots in Algorithm 1.

**Algorithm 6 Energy-performance tradeoff scheduling algorithm**

1. schedule tasks via the ETF scheduling algorithm 5
2. scale down PE’s voltages for critical jobs with Algorithm 7
3. scale down PE’s voltages for non-critical jobs with Algorithm 1

The Algorithm 7 firstly extends the critical time slots. Assume $job_n$ is a critical job and it is executed on $pe_k$. It has been proved in [46] that distributing the free slack time “evenly” (proportional to the original critical time) is optimal as the power consumption is a convex function of PE frequency. Therefore $job_n$’s slack time can be calculated as:

$$\text{slack} = job_n \cdot t^0 \times \eta$$  \hspace{1cm} (23)

where:

- $job_n$’s execution time when $pe_k$ is operated with $f_{max}$.
- $\eta$ is the agreed extension of parallel task’s execution time (see Fig. 5).

$pe_k$’s operating frequency can be scaled to $pe_k \cdot f^{op}$.

$$pe_k \cdot f^{op} = f_{max} \times \frac{job_n \cdot t^0}{job_n \cdot \text{slack}}.$$  \hspace{1cm} (24)

10. Algorithm complexity analysis

In this section, we present an analysis on the time complexity of the algorithms discussed above.

**Algorithm 7 Algorithm of voltage scaling for all time slots**

1. BEGIN
2. FOR each PE $pe_k$ DO
3. FOR each time slot in $pe_k$’s Gantt chart DO
4. IF $pe_k$ executes a critical job $job_n$ THEN
5. calculate its $job_n$’s slack time as Eq. (23)
6. scale $pe_k$’s frequency to $pe_k \cdot f^{op}$ as Eq. (24).
4. ENDFOR
3. FOR each time slot in $pe_k$’s Gantt chart DO
4. IF $pe_k$ is idle or it executes a communication phase THEN
5. scale down $pe_k$’s operating frequency to lowest
6. ENDFOR
7. IF $pe_k$ executes a non-critical job $job_n$ THEN
8. calculate $job_n$’s slack as Eq. (21).
9. scale $pe_k$’s frequency to $pe_k \cdot f^{op}$ as Eq. (22).
10. ENDFOR
11. ENDIF
12. ENDFOR
13. END

10.1. Analysis of the PTAC algorithm

10.1.1. Algorithm 1

Algorithm 1 scales the supply voltage of a PE. Assuming we have $K$ PE’s, with $t$ time slots, line 2 will occur at most $K$ times, where as the inner loop starting at line 3 will occur $t$ times. The operations from lines 4 to 10 are constant time operations, thus the upper bound of this algorithm is $O(Kt)$.

10.1.2. Algorithm 2

This algorithm forms the task clusters. Line 2 is executed $|J|$ times. The sorting in line 3 can be done in $|J|\lg |J|$ time via quicksort. Lines 5 and 6 are constant time operations, each of which is part of a loop of $|J|$ iterations. Line 7 issues a call to Algorithm 3 when two clusters are merged. Since initially, each Task forms a cluster, we have $C$ clusters and a total of $T$ tasks. At most, Algorithm 3 will be called $CT$ times. $O(|E| + |E|\lg |E| + CT * A3)$ where $A3$ represents the complexity of the $b$ _level_ calculation, or Algorithm 3.

10.1.3. Algorithm 3

This algorithm computes the $b$ _level_ for a task. This algorithm is called by Algorithm 2. The sorting of line 2 can be done in $|J|\lg |J|$ time. Line 3 is an initialization that occurs $|J|$ times. Lines 4 and 6 are a double loop, however each loop inner loop only iterates through a job J’s children. Thus, the total number of iterations for lines 4–13 occurs $|E|$ times. Thus, Algorithm 3’s complexity is $O(|J|\lg |J| + |J| + |E|)$. Thus, our loose upper bound for the PTAC algorithm is $O(|E| + |E|\lg |E| + C(|J|\lg |J| + |J| + |E|))$.

10.2. Analysis of the PALS algorithm

10.2.1. Algorithm 4

Algorithm 4 simply executes Algorithms 1 and 5. For example, Algorithm 5 will be executed $T$ times, where $T$ represents the total number of tasks.

10.2.2. Algorithm 5

This algorithm schedules the jobs of a task on to the PEs. Lines 1–4 are simply descriptions, or comments. Lines 6–8 compute the priority for each job in the task, and execute $N$ times, where $N$ represents the number of jobs in the task. Line 9 is also execute $N$ times, and simply adds jobs to a list. The sorting of the jobs in line 10 can be done in $O(N\lg N)$ time. Line 11 is linear complexity,
like line 9, and simply places the PEs into a list. This is done K times. The sorting in line 12 can be done in K lg K time. The loop in lines 13–23 loops through each job in the list. This is done N times. Each operation between lines 13–23 can be considered to be done in constant time, for example, retrieving a job from the list in line 15 is constant. The complexity for Algorithm 5 is thus \( O(N + N lg N + K + K lg K) \).

### 10.2.3. Algorithm 6

Algorithm 6 represents the energy-performance tradeoff algorithm. Line 1 makes \( T \) calls to Algorithm 5, where \( T \) represents the number of tasks. Likewise, in lines 2 and 3, Algorithm 6 calls Algorithm 7 and Algorithm 1 \( T \) times. Thus, the complexity of Algorithm 6 is \( O(T(Kt + N + N lg N + K + K lg K + A7)) \) where \( A7 \) represents the complexity of Algorithm 7.

### 10.2.4. Algorithm 7

Algorithm 7 scales down the voltage for the critical path, thus increasing the execution time of the task as a whole. The outer loop on line 2 is executed \( K \) times for the \( K \) PEs. The first inner loop on line 3 gets executed \( t \) times, where \( t \) represents the number of time slots in the Gantt chart. Lines 4, 5, and 6 are constant time operations.

The second inner loop has \( t \) loops, for each time slot in the Gantt chart. Thus, the complexity for the PALS algorithm is \( O(Kt + Kt) \), or simply \( O(Kt) \).

### 11. Performance study with simulation

We make a simulation study on the proposed best-effort scheduling algorithm and energy-performance tradeoff scheduling algorithm. Several task sets are generated with the Synthetic DAG generation tool [47]. We simulate a cluster with multiple Turion MT-34 processors, whose operating points are shown in Table 1.

In this simulation for best-effort scheduling, we are interested in how much energy is saved given various parallel tasks and PE numbers in the cluster. We define the resource competition to execute a parallel task, \( \zeta(T) \), in a cluster as follows:

\[
\zeta(T) = \frac{N}{P}
\]

where, \( T \) is the parallel task, \( N \) is the job number of \( T \), and \( P \) is the PE number for executing \( T \). Resource competition shows the task execution situation, like how many precedences exist between jobs, how many jobs are scheduled, and how many jobs are executed on each PE.

The PATC and the PALS can achieve up to 39.7% and 44.3% energy saving respectively in the simulation. Table 2 compares our algorithm with other energy aware DAG scheduling algorithms in term of max energy saving. EADUS & TEBUS [29] use the duplication strategies for scheduling DAG based parallel tasks in a cluster to reduce power consumption. However, EADUS & TEBUS do not use DVFS to reduce energy consumption, thus leading less energy savings. Compared with LEneS [23], Energy Reduction Algorithm [33], and ECS [32], the PATC and PALS can achieve more energy saving as

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<th>Energy aware DAG scheduling algorithm</th>
<th>Maximum energy saving (%)</th>
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<tr>
<td>EADUS &amp; TEBUS [29]</td>
<td>16.8</td>
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<td>Energy reduction algorithm [33]</td>
<td>25</td>
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<td>LEneS [23]</td>
<td>28</td>
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<td>ECS [32]</td>
<td>38</td>
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<td>PATC</td>
<td>39.7</td>
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<td>PALS</td>
<td>44.3</td>
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Fig. 6 shows the energy savings when running the PALS algorithm in different scenarios of numbers of PEs and resource competition. For a close view, Figs. 7 and 8 shows two special cases of (1) Energy savings when running the PALS algorithm with different scenarios of resource competition and PE number is set as 50; (2) Energy savings when running the PALS algorithm with different PE numbers and resource competition is set as 6. From the above figures we can see that the energy saved increases as the number of PEs increases. This can be explained as follows: when the number of PEs increases, intuitively there are less jobs executed in a PE, then the jobs have more of a chance to scale their execution time
and PE supply voltages. If we fix the number of PEs, the energy saving firstly increases, achieves its maximum value, and then it decreases. This can be explained by the fact that the percentage of jobs on the critical path firstly increases then decreases. The length of the critical path gives the limit that non-critical jobs can extend to.

In the simulation for energy-performance tradeoff scheduling, we are more interested in the relationship between the energy saved and the extended task execution time, as shown in Fig. 9. From Fig. 9 we can see that:

- When the makespan extension increases, the energy savings also increase.
- Then energy savings increase much when the makespan extension is less than 30%.

These observations can conclude that the green SLA negotiation is feasible. When users pay additional tolerant task execution time, which is less than 30%, less than 70% energy saving can be achieved. This is a win–win game.

12. Conclusion and future work

Recently, the need for efficient algorithms to minimize wasted server energy has become increasingly important. Dynamic voltage and frequency scaling (DVFS) technique has proven to be a highly effective technique to achieve low power consumption for high performance computing by dynamically scaling processor speed. We develop our research on minimizing energy for precedence-constrained parallel task execution. This paper proposes two scheduling algorithms in DVFS-enabled clusters for executing parallel tasks: the PATC and PALS. The proposed algorithms search the slack time for non-critical jobs without increasing scheduling length. We also develop a green SLA-based mechanism to reduce energy consumption by return users tolerant increased scheduling makespan. The proposed scheduling algorithm is examined via a simulation study. Test results show that the scheduling algorithm is efficient to reduce the power consumption of a DVFS-enabled cluster. Future work includes the deployment of the power aware scheduling algorithm in some real applications, for example, the sparse Cholesky decomposition.

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SWaT (Space, Watts and Performance) Metric, Web Page


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